Name: DR. LOVELESS

Some info about yourself (optional)
Town/High School you are from: YAKIMA, EISENHOWER HS
Intended major: MATH & COMPUTER SCIENCE
An interest/hobby: FAMILY, TENNIS, BOOKS, MOVIES, BASEBALL, SOCCER

Test Prep 1

This test prep includes the two problems below. You get 10 minutes. If you complete the front and have more time, you can get one extra point by correctly doing the integration by parts on the back of this page.

1. A town currently has 2100 people The birth/death rate is proportional to the population size with a relative growth rate of $k = 0.03$. In addition, 100 people/year are immigrating into the city from elsewhere. Let $P(t)$ be the number of people in the city in t years from now. Give the differential equation and initial condition for this situation (Do NOT solve, just write equations).

$\frac{dP}{dt} = 0.03P + 100$

$P(0) = 2100$

$0.03P$ = "people added per year by births" (3% of population)
AND
100 $\frac{\text{people}}{\text{year}}$ added by immigration
TOTAL ADDED PER YEAR $= 0.03P + 100$

2. Find the numbers A, B, and C that make the following true for all valid values of x

$$x^2(x+3)\left[\frac{6}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}.\right]$$

Then integrate:

$$\int \frac{6}{x^2(x+3)}\,dx$$

NO x^2, NO x

$6 = Ax(x+3) + B(x+3) + Cx^2 = Ax^2 + 3Ax + Bx + 3B + Cx^2$

COMPARE COEFFICIENTS: COEF. OF x^2: $0 = A + C$ ⟹ $A = -2/3$ ($C = 2/3$)
COEF. OF x: $0 = 3A + B$
CONSTANT COEF: $6 = 3B \rightarrow B = 2$

$$\int \frac{-2/3}{x} + \frac{2}{x^2} + \frac{2/3}{x+3}\,dx = -\frac{2}{3}\ln|x| - \frac{2}{x} + \frac{2}{3}\ln|x+3| + C$$

Optional Extra Credit (1 pt): Evaluate $\int x^2 \ln(x)\, dx$

$$\int u\,dv = uv - \int v\,du$$

$u = \ln(x)$ $\quad dv = x^2\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = \frac{1}{3}x^3$

$$\int x^2 \ln(x)\,dx = \frac{1}{3}x^3\ln(x) - \int \frac{1}{3}x^2\,dx$$

$$= \boxed{\frac{1}{3}x^3\ln(x) - \frac{1}{9}x^3 + C}$$